

Johnson-Holmquist 2 Material Model

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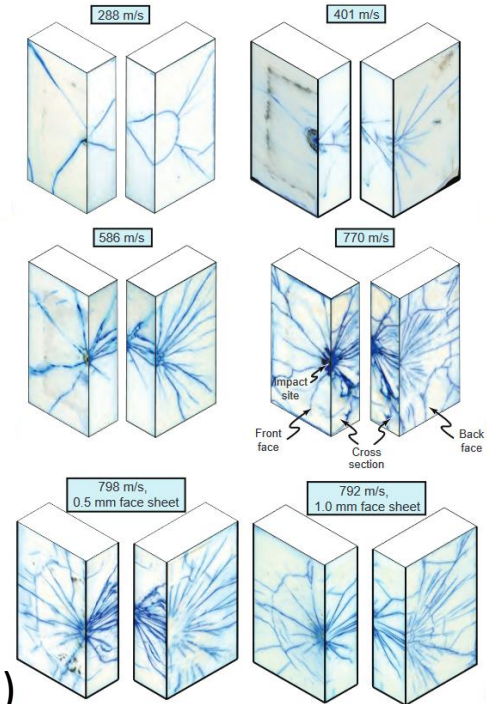
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Johnson-Holmquist 2 (JH-2)

- Phenomenological, strain-rate, and pressure-dependent softening plasticity model
- Challenges:
 - Mesh size dependency
 - Time step dependency
 - Ambiguities in material properties



(Compton et al., 2012)

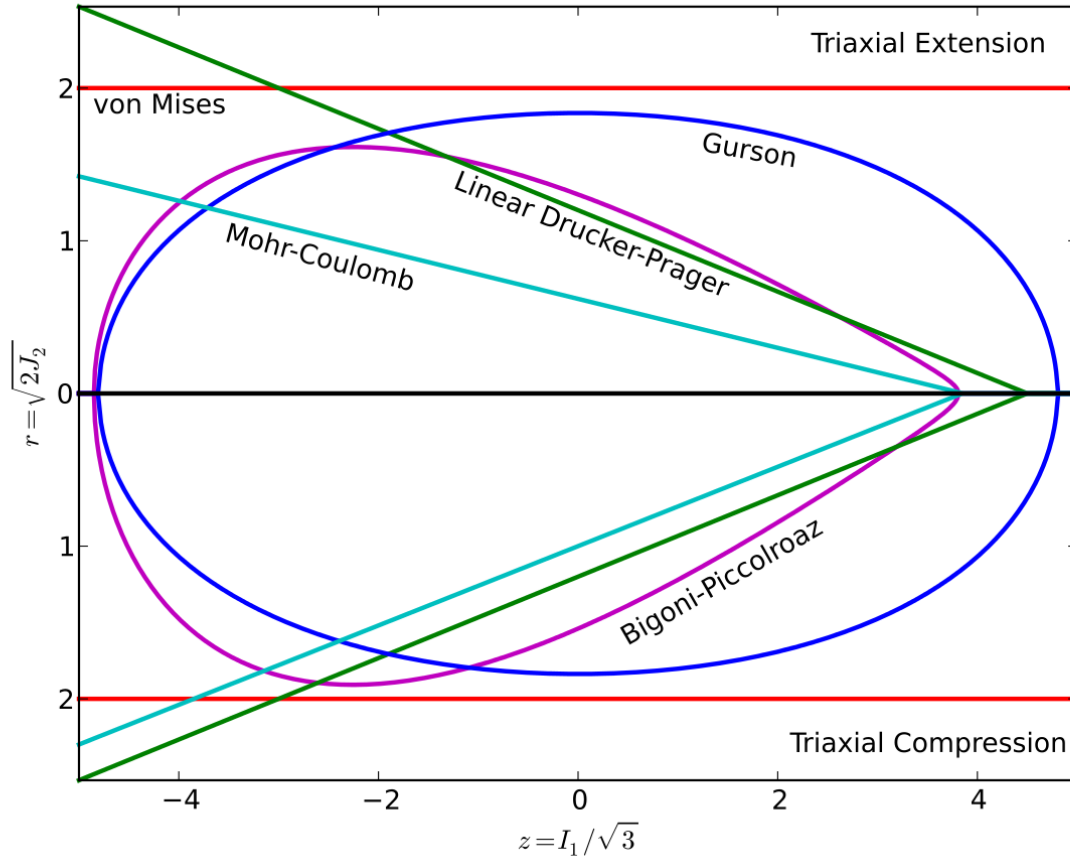
Material properties

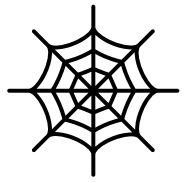
	B4C	SiC	AlN	Al2O3	Silica Float Glass
Reference	[9]	[12]	[10]	[8]	[11]
Density (kg/m ³)	2510	3163	3226	3700	2530
Shear Modulus (GPa)	197	183	127	90.16	30.4
Strength Constants					
A	0.927	0.96	0.85	0.93	0.93
B	0.7	0.35	0.31	0.31	0.088
C	0.005	0.0	0.013	0.0	0.003
M	0.85	1.0	0.21	0.6	0.35
N	0.67	0.65	0.29	0.6	0.77
Ref Strain Rate (EPSI)	1.0	1.0	1.0	1.0	1.0
Tensile Strength (GPa)	0.26	0.37	0.32	0.2	0.15
Normalized Fracture Strength	0.2	0.8	NA	NA	0.5
HEL (GPa)	19	14.567	9	2.79	5.95
HEL Pressure (GPa)	8.71	5.9	5	1.46	2.92
HEL Vol. Strain	0.0408		0.0242	0.01117	
HEL Strength (GPa)	15.4	13.0	6.0	2.0	4.5
Damage Constants					
D1	0.001	0.48	0.02	0.005	0.053
D2	0.5	0.48	1.85	1.0	0.85
Equation of State					
K1 (GPa) (Bulk Modulus)	233	204.785	201	130.95	45.4
K2 (GPa)	-593	0	260	0	-138
K3 (GPa)	2800	0	0	0	290
Beta	1.0	1.0	1.0	1.0	1.0

Table 1 Constitutive constants for ceramic materials

(Cronin et al., 2004)

Meridional Profile of Several Plasticity Models





Johnson-Holmquist 2 (JH-2)

$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*)$	Equivalent stress for ceramic
$\sigma_i^* = A(P^* + T^*)^n(1 + C \ln \dot{\epsilon}^*)$	Intact behavior
$\sigma_f^* = B(P^*)^m(1 + C \ln \dot{\epsilon}^*)$	Fractured behavior

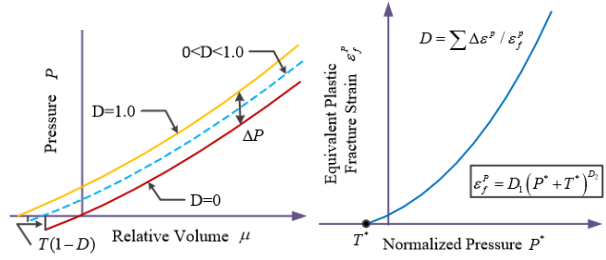
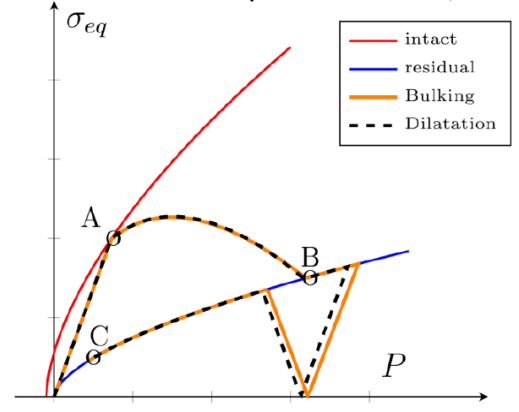
Equation of state

$\mu = \frac{\rho}{\rho_0} - 1$	$P = K_1\mu + K_2\mu^2 + K_3\mu^3 + \Delta P_f$	Compression
	$\Delta P_f = -K_1\mu_f + \sqrt{(K_1\mu_f)^2 + 2\beta K_1\Delta U}$	Bulking pressure
Volumetric strain	$P = K_1\mu$	Tension

Damage

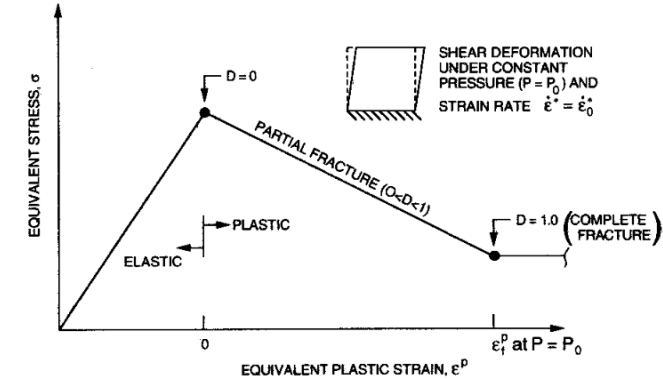
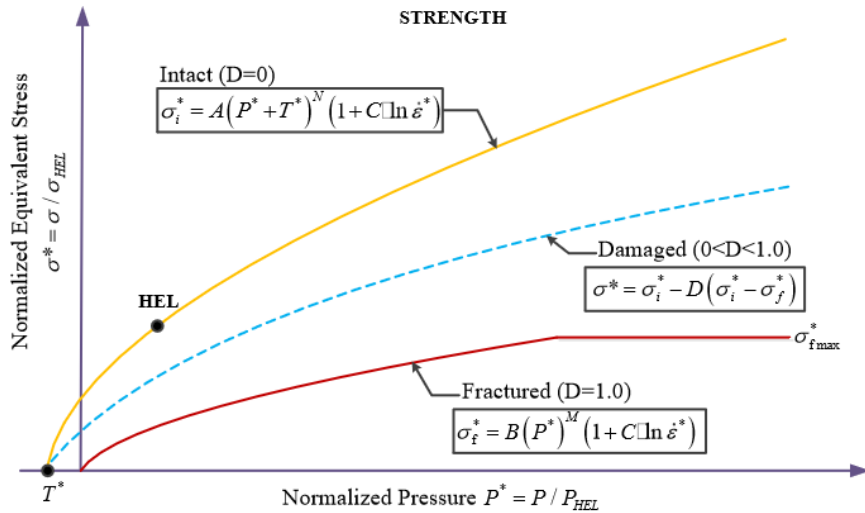
$D = \sum \frac{\Delta \epsilon^p}{\epsilon_f^p}$	$\epsilon_f^p = d_1(p^* + t^*)^{d_2}$	Equivalent plastic strain to fracture under constant pressure
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(Simons et al., 2016)



(Wang et al., 2018)

JH-2 Stress-Strain

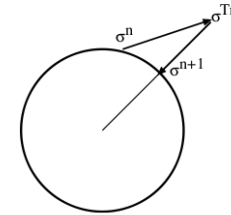
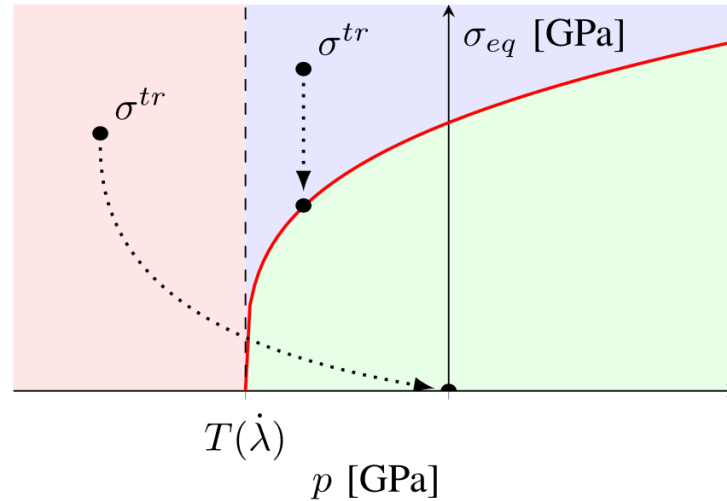


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Figure 2. Strength, damage, and fracture under a constant pressure and strain rate

(Johnson & Holmquist, 1994)

Radial return



(Hallquist, 2006)

(Simons et al., 2018)

Fig. 2. Return mapping for the JH2 model visualized. Any trial state in the green stress space remains elastic, in the blue stress space regular return mapping is applied while the red domain requires apex return.

Table 1: Details on the model implementation.

Step	Description	Equation
1	Strain update at current time step	$\varepsilon_{ij}^{t+\Delta t} = \varepsilon_{ij}^t + \Delta\varepsilon_{ij}$
2	Compute strain rates	$\dot{\varepsilon}_{ij}^{t+\Delta t} = \Delta\varepsilon_{ij}/\Delta t$
3	Compute strain increments	$\Delta\sigma_{ij} = C\Delta\varepsilon_{ij}$
4	Compute trial stresses at current time step	$trial\ \sigma_{ij}^{t+\Delta t} = \sigma_{ij}^t + \Delta\sigma_{ij}$
5	Split total trial stresses into deviatoric and hydrostatic stresses	
5.1	Hydrostatic stress	$\sigma_H^{t+\Delta t} = \frac{1}{3}(trial\ \sigma_{11}^{t+\Delta t} + trial\ \sigma_{22}^{t+\Delta t} + trial\ \sigma_{33}^{t+\Delta t})$
5.2	Deviatoric stresses	$S_{ij}^{trial} = trial\ \sigma_{ij}^{t+\Delta t} - \sigma_H^{t+\Delta t}\delta_{ij} \quad (1)$
6	Compute total effective strain rate	$\dot{\varepsilon} = \sqrt{\frac{2}{3}}\sqrt{\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + \varepsilon_{zz}^2 + \frac{1}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)}$
7	Compute equivalent $\bar{\sigma}$ stress using Eq. (10)	Eq. (10)
8	Compute normalized yield stress	Eq. (4), (5), (1)
9	Check for yielding	Eq.(9)
9.1	If $F(\bar{\sigma}) > 0$ return the deviatoric stresses to the yield surface using the radial return algorithm	$S_{ij} = (\sigma_{HEL} \cdot \sigma^*)/S_{ij}^{trial}$
9.2	Compute the plastic strain increment $\Delta\varepsilon_p$ using Eq. (17)	Eq. (17)
9.3	Compute the plastic strain to fracture ε_f^p using Eq. (7)	$\varepsilon_f^p = D_1(P^* + T^*)D_2$
9.4	Update the damage variable D	$D^{t+\Delta t} = D^t + (\Delta\varepsilon_p)/\varepsilon_f^p$
10	Compute the compressibility factor	$\mu^{t+\Delta t} = \ln(\varepsilon_\nu^{t+\Delta t} + 1)$
10.1	If $\mu^{t+\Delta t} > 0$ compute pressure from equation of state Eq. (20)	$P = K_1\mu + K_2\mu^2 + k_3\mu^3 + \Delta P$
10.2	Else if $\mu^{t+\Delta t} < 0$,	$P^{t+\Delta t} = K_1\mu^{t+\Delta t}$
10.3	If $D^{t+\Delta t} > 0$ compute energy loss due to damage using Eqs. (21) and (22) with	$U_D^t = (\sigma_i^* - D^t(\sigma_i^* - \sigma_f^*) \cdot \sigma_{HEL})^2 / (6G)$ $U_D^{t+\Delta t} = (\sigma_i^* - D^{t+\Delta t}(\sigma_i^* - \sigma_f^*) \cdot \sigma_{HEL})^2 / (6G)$
10.4	And compute the pressure increment $\Delta P^{t+\Delta t}$ using Eq. (23)	$P^{t+\Delta t} = \sigma_H^{t+\Delta t} + \Delta P^{t+\Delta t}$
11	Compute new total stress	$\sigma_{ij}^{t+\Delta t} = S_{ij} + P^{t+\Delta t}\delta_{ij}$
12	Go to step 1	

$$\bar{\sigma} = \sqrt{3} \left[\frac{1}{2} (S_{xx}^2 + S_{yy}^2 + S_{zz}^2) + S_{xy}^2 + S_{yz}^2 + S_{zx}^2 \right]^{\frac{1}{2}}$$

$$\sigma_i^* = A(P^* + T^*)^N (1 + C \ln(\dot{\varepsilon}^*))$$

$$\sigma_f^* = B(P^*)^M (1 + C \ln(\dot{\varepsilon}^*))$$

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*)$$

$$F(\bar{\sigma}) = \bar{\sigma} - \sigma_{HEL} \cdot \sigma^*$$

$$\Delta\varepsilon_p = \frac{a^T C \dot{\varepsilon} \Delta T}{\sqrt{2} a^T C a} \sqrt{(S_{xx} - S_{yy})^2 + (S_{xx} - S_{zz})^2 + (S_{yy} - S_{zz})^2 + 6[S_{xy}^2 + S_{yz}^2 + S_{zx}^2]}$$

(Bürger & Donadon, 2009)

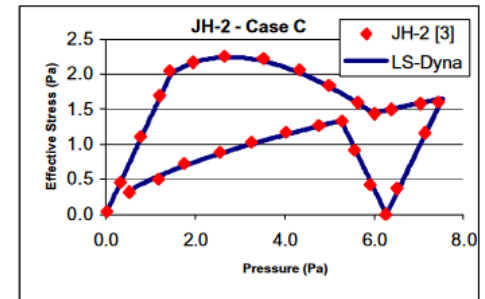
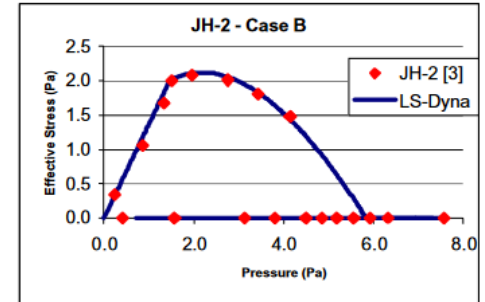
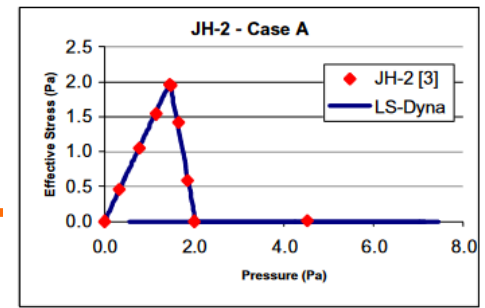
(1) $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ otherwise

Algorithm

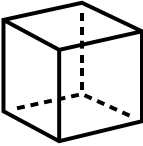
(Cronin et al., 2004)

	Case A	Case B	Case C
Density (kg/m ³)	3700	3700	3700
Shear Modulus (GPa)	90.16	90.16	90.16
Strength Constants			
A	0.93	0.93	0.93
B	0	0	0.31
C	0	0	0
M	0	0	0.6
N	0.6	0.6	0.6
Ref Strain Rate (EPSI)	1.0	1.0	1.0
Tensile Strength (GPa)	0.2	0.2	0.2
Normalized Fracture Strength	0	NA	NA
HEL (GPa)	2.79	2.79	2.79
HEL Pressure (GPa)	1.46	1.46	1.46
HEL Vol. Strain	0.01117	0.01117	0.01117
HEL Strength (GPa)	2.0	2.0	2.0
Damage Constants			
D1	0	0.005	0.005
D2	0	1	1.0
Equation of State			
K1 (GPa) (Bulk Modulus)	130.95	130.95	130.95
K2 (GPa)	0	0	0
K3 (GPa)	0	0	0
Beta	1.0	1.0	1.0

Table 2 Constitutive constants for validation cases

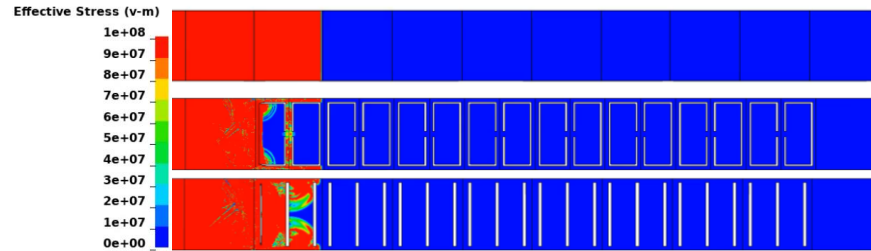


Results

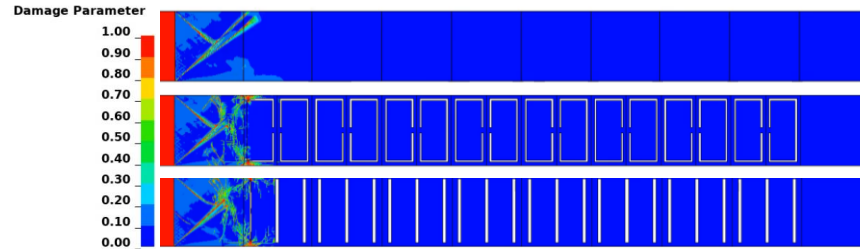


- 20 mm projectile
- Plate impact
- $v = 30, 100, 300$ m/s
- Top and bottom roller
- Right side free

$v = 300$ m/s
von-Mises stress

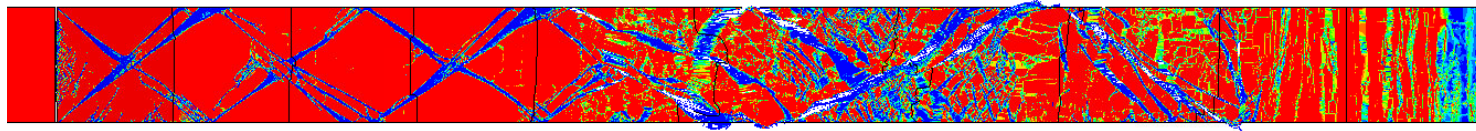


Damage parameter

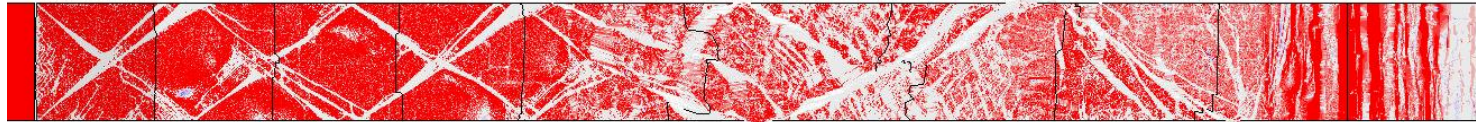


Results

von-Mises stress

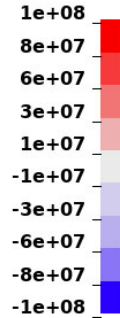
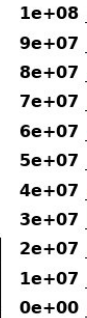


Pressure

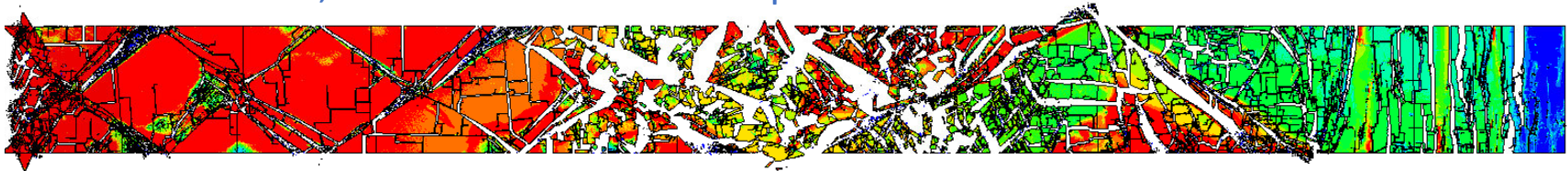


Effective Stress (v-m)

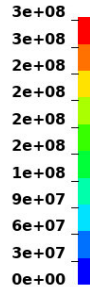
Pressure



von-Mises stress, elements with effective plastic strain > 0.015 removed



Effective Stress (v-m)



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