Discontinuous Galerkin Finite Element Methods

This course is intended to serve as a sequel to an introductory finite element method (FEM) course where conventional (continuous) FEM method is covered. The main difference of Discontinuous Galerkin (DG) methods to continuous FEMs the weak enforcement of jump conditions on the boundary of the elements. DG methods generally are more stable and perform better for dynamic problems involving shocks and other discontinuities. In this course we cover:

1. Overview of balance laws, strong and weak forms, weighted residual and weak statement of FEM formulation.
2. Brief classification of elliptic, parabolic, and hyperbolic partial differential equations (PDEs).
3. Mathematical statement of systems of conservation laws (hyperbolic systems): characteristics; solution feature such as shocks, rarefaction waves, and contact
4. Rankine-Hugoniot jump conditions for conservation laws; Exact and some approximate Riemann solution schemes.
5. Differential forms (exterior calculus) to objectively express and combine space and time quantities.
6. Finite element formulation for DG methods.
7. Computational geometry aspects of DG methods (mesh smoothing, h-, p-, and hp-adaptive operations, moving boundaries, etc.).
8. Object-oriented design and implementation of FE methods.

More than half of the class covers the mathematical and computational background needed for DG methods. In the remainder of the class (items 7 and 8) we cover FE adaptive concepts (geometry and physics) and implement new conversation law systems with Spacetime Discontinuous Finite Element method. Some sample systems that students could formulate and implement include: acoustics equation, simple advection-diffusion-reaction equations, elastodynamics, and shallow water equations.

Course Objectives

1. Provide sufficient mathematical background to read the current literature on DG methods.
2. In depth understanding of the process of FE formulation starting from the balance laws. Students would be able to formulation various DG and Continuous FE formulations on their own.
3. Familiarize students with differential forms and their advantages over conventional tensorial notation.

4. Relate theory to practical applications in computational science and engineering by exposure and implementation of highly modular object oriented finite element codes.
5. Familiarize students with geometric meshing aspects of finite element methods.

Homework	10%
Midterm exams	15%
Term projects	75%

Recommended Text

- W.H. Fleming, Functions of Several Variables, Addison-Wesley, Reading, MA, 1964.
- S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 1994.
- R. J. Leveque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2003.