Istanbul Technical University

Graduate School of Science Engineering and Technology

MKC517E
Special Topics in Solid Mechanics

Final

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A flat plate with a through-thickness crack (Fig. 1.8) is subject to a 100 MPa (14.5 ksi) tensile stress and has a fracture toughness (K_{IC}) of $50\,\mathrm{MPa}\sqrt{\mathrm{m}}$ ($45.5\,\mathrm{ksi}\sqrt{\mathrm{in}}$). Determine the critical crack length for this plate, assuming the material is linear elastic.

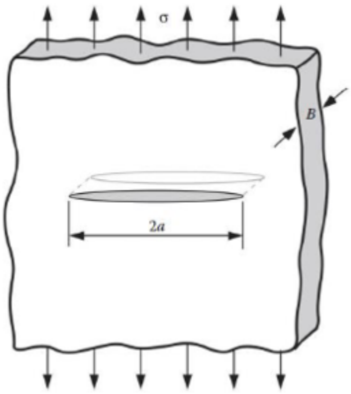

Figure 1: Finite width plate with a through crack at the center of plate.

$$\mathrm{K_{IC}} = \mathrm{K_I} = \sigma\sqrt{\pi \mathrm{a_c}} \tag{1.1}$$
$$50 = 100\sqrt{\pi \mathrm{a_C}} \tag{1.2}$$
$$\mathrm{a_c} = 79.58\ \mathrm{mm} \tag{1.3}$$

So total crack length $2a_c$ equals to 159.15 mm.

Compute the critical energy release rate $\mathcal{G}_c$ of the material for E = 207 000 MPa

$$\mathcal{G} = -\frac{d\Pi}{d\mathrm{A}} = \frac{\pi\sigma^2 a}{E} \tag{1.4}$$

$$\mathcal{G}_\mathrm{c} = \frac{\mathrm{K_{IC}}}{\mathrm{E}} \tag{1.5}$$

So,

$$\frac{50^2}{207000} = 0.012\,077\,\mathrm{MPa\cdot m} = 0.012\,077\,\mathrm{J\cdot mm^{-2}} \tag{1.6}$$
$$\mathcal{G}_\mathrm{c} = 12.077\,\mathrm{kJ\cdot m^{-2}} \tag{1.7}$$

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A material exhibits the following crack growth resistance behavior:

$$R = 6.95\,(a - a_0)^{0.5} \tag{2.1}$$

where a_0 is the initial crack size. R has units of $\mathrm{kJ \cdot m^{-2}}$ and crack size in millimeters. Elastic modulus of this material E = 207 000 MPa. Consider a wide plate with a through crack ($a \ll W$) that is made from this material.

If this plate fractures at 138 MPa, compute the following:

The half crack size at failure (a_c)

The conditions for stable crack growth can be expressed as follows:

$$\mathcal{G} = R \tag{2.2}$$

and

$$\frac{d\mathcal{G}}{da} \leq \frac{dR}{da} \tag{2.3}$$

Unstable crack growth occurs when

$$\frac{d\mathcal{G}}{da} > \frac{dR}{da} \tag{2.4}$$

$$\mathcal{G} = \frac{\pi\sigma^2 a_c}{E} = 6.95\,(a_c - a_0)^{0.5} \tag{2.5}$$

$$\frac{d\mathcal{G}}{da} = \frac{dR}{da} \tag{2.6}$$

$$\frac{\pi\sigma^2}{E} = 3.457\,(a_c - a_0)^{-0.5} \tag{2.7}$$

$\sigma = 138\,\mathrm{MPa}$

$$\frac{\pi 138^2}{207000} = 3.457\,(a_c - a_0)^{-0.5} \tag{2.8}$$

$$a_c - a_0 = 144.56\,\mathrm{mm} \tag{2.9}$$

Substituting into Eq. 2.5

$$a_c = 289.11\,\mathrm{mm} \tag{2.10}$$

The amount of stable crack growth (at each crack tip) that precedes failure ($a_c - a_0$), Eq. 2.9

If this plate has an initial crack length ($2a_0$) of 50.8 mm and the plate is loaded to failure, compute the following:

Stress at failure

Using Equations. 2.5 and 2.7

$$a_c = 2(a_c - a_0) \tag{2.11}$$

If $a_0 = 25.4\,\text{mm}$, we get $a_c = 50.8\,\text{mm}$. Using Eq. 2.5

$$\mathcal{G} = \frac{\pi\sigma^2 50.8}{207000} = 6.95(25.4)^{0.5} \tag{2.12}$$

Thus, stress at failure

$$\sigma = 213.15\,\text{MPa} \tag{2.13}$$

The half crack size at failure $a_c = 50.8\,\text{mm}$

The stable crack growth at each crack tip $a_c - a_0 = 25.4\,\text{mm}$