ISTANBUL TECHNICAL UNIVERSITY

GRADUATE SCHOOL OF SCIENCE ENGINEERING AND TECHNOLOGY

MKC517E
SPECIAL TOPICS IN SOLID MECHANICS

Homework 3

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Center crack in a finite width plate (TC01 crack scenario, Figure 1) with 1000 mm width and 5 mm thickness is considered using the load scenarios given in Tables 2 and 3. Loads are applied as tensile/compression and given in Table 1. Cycle at which panel fails for initial crack length of 0.2 mm is to be determined using two different stress scale factors ($S_0 = 1$ and $S_0 = 0.7$). Material is 2024-T3 CLAD PLT & SHT; L-T; LA & HHA.

Table 1: Applied loads.

Load name	S_{max} [MPa]	S_{min} [MPa]
L_1	250	0
L_2	250	-250
L_3	350	0
L_4	350	-350

Table 2: Load scenarios (a to d).

Load Scenario	**Load name**	**Number of cycles in a block**	**Number of blocks**	**Retardation model**
a	L_1	1000	2500	Non-interactive
b	L_1	1000	2500	MGW
c	L_2	1000	2500	Non-interactive
d	L_2	1000	2500	MGW

Table 3: Load scenarios (e to f).

	First load		Second load			
Load Scenario	Load name	Number of cycles	Load name	Number of cycles	Number of blocks	Retardation model
e	L_1	50000	L_3	1	50	MGW
f	L_1	50000	L_4	1	50	MGW
g	L_1	50000	L_3	10	50	MGW
h	L_1	50000	L_4	10	50	MGW

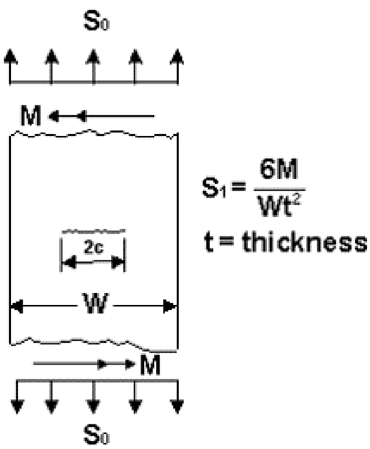

Figure 1: Finite width plate with a through crack at the center of plate (TC01).

In this study, effect of retardation is examined using different load scenarios. No retardation (Non-interaction) crack growth model is compared with the Modified Generalized Willenborg (MGW) retardation model with three different retardation parameter (ϕ_0). Forman crack growth rate equation is used for growth-rate calculations.

Crack growth rate equation is given by:

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]\frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_c}\right)^q} \tag{0.1}$$

where N is the number of applied fatigue cycles, a is the crack length, R is the stress ratio, ΔK is the stress intensity factor range; C, n, p, and q are empirically derived constants, ΔK_{th} is the the threshold stress intensity factor, ΔK_c is the the critical stress intensity factor and f is the crack opening function.

The Generalized Willenborg model, utilizes a residual stress intensity, K_R , which determines the effective stress ratio due to a load interaction as follows:

$$R_{\text{ eff}} = \frac{K_{\min} - K_R}{K_{\max} - K_R} = \frac{K_{\min,kf}}{K_{\max,\text{ eff}}} \tag{0.2}$$

R_{eff} is used instead of the actual stress ratio within the crack growth equation and has the effect of retarding the crack growth.

The retardation for a given applied cycle of loading depends on the loading and the extent of crack growth into the overload plastic zone. Gallagher expressed the Willenborg residual stress-intensity factor as:

$$K_R^W = K_{\max}^{OL}\left(1 - \frac{\Delta a}{Z_{OL}}\right)^{\frac{1}{2}} - K_{\max} \tag{0.3}$$

where $K_{\max}^{OL}$ is the maximum stress intensity for the overload cycle, and Δa is the crack growth between the overload cycle and the current cycle. The overload plastic zone size Z_{OL} is given by:

$$Z_{OL} = \frac{\pi}{8}\left(\frac{K_{\max}}{\alpha_g \sigma_{ys}}\right)^2 \tag{0.4}$$

The constraint factor α_g is taken from a fit developed by Newman and is given by:

$$\alpha_g = 1.15 + 1.4e^{-0.95\left(\frac{K_{\max}}{\sigma_{ys}\sqrt{t}}\right)^{1.3}} \tag{0.5}$$

K_R^W represents the difference between the stress intensity required to produce a plastic zone equal to Z_{OL} and the current maximum applied stress intensity $K_{\max}$. In the original development, retardation is considered to occur if $K_R^W > 0$. In the Generalized Willenborg model, a modified residual stress-intensity K_R is used, related to K_R^W by

$$K_R = \phi K_R^W \tag{0.6}$$

where,

$$\phi = \frac{1 - \frac{\Delta K_{th}}{\Delta K}}{(R_{SO} - 1)} \tag{0.7}$$

R_{SO} is the shut-off value of the stress ratio K_{max}^{OL}/K_{max}, When this value is exceeded, $K_{max,\text{ eff}}$ is set equal to $\Delta K_{th}/(1 - R)$ and crack growth is arrested. No special consideration is given to multiple overloads and their effect is taken to be the same as that for a single overload.

The MGW model extends the Generalized Willenborg load interaction model by taking into account the reduction of retardation effects due to underloads. The MGW model (like the Generalized Willenborg), utilizes a residual stress intensity, K_R , which determines the effective maximum and minimum stress due to a load interaction. The equations are:

$$\begin{aligned} K_{\max}^{\text{ef}} &= K_{\max} - K_R \\ K_{\min}^{\text{eff}} &= \text{Max}\left\{(K_{\min} - K_R), 0\right\}, \quad &&\text{for } K_{\min} > 0 \\ &= K_{\min} &&\text{for } K_{\min} \leq 0 \end{aligned} \tag{0.8}$$

The factor ϕ in that equation is now given by:

$$\begin{aligned} \phi &= 2.523\phi_0 / \left(1.0 + 3.5\,(.25 - R_U)^6\right) \quad &, R_U < 0.25 \\ &= 1.0 &, R_U \geq 0.25 \end{aligned} \tag{0.9}$$

The parameter ϕ_0 is the value of ϕ for $R_U = 0$. Parameter ϕ_0 is a material dependent parameter that can be determined, ideally, by conducting a series of typical aircraft spectrum tests. The value of ϕ_0 ranges typically from 0.2 to 0.8. Formulation is taken from Nasgro manual.

For given load scenario ϕ_0 is not affecting the results since $R_U \geq 0.25$. However for stress scale factor $S_0 = 0.7$ and $\phi_0 = 0.8$, overload value of 350 MPa (scales down to 245 MPa with the scale factor), results in trapped crack. Which means that crack does not grow after the first block. Results of crack growth calculations are given in Tables 4 and 5.

Table 4: Results for $S_0 = 1$ for different retardation parameters.

	ϕ_0	Crack Size	Total Cycles
a	n/a	25.6762	42926.804
b	0.2	25.6947	42926.824
b	0.5	25.6947	42926.824
b	0.8	25.6947	42926.824
c	n/a	25.7447	25138.019
d	0.2	25.6939	25138.031
d	0.5	25.6939	25138.031
d	0.8	25.6939	25138.031
e	0.5	25.6895	42930.078
f	0.5	25.6895	42930.078
g	0.5	25.6895	42930.078
h	0.5	25.6895	42930.078

Table 5: Results for $S_0 = 0.7$ for different retardation parameters.

	ϕ_0	Crack Size	Total Cycles
a	n/a	52.0244	186117.92
b	0.2	51.849	186118.14
b	0.5	51.849	186118.14
b	0.8	51.849	186118.14
c	n/a	51.8624	98629.948
d	0.2	51.9077	98629.966
d	0.5	51.9077	98629.966
d	0.8	51.9077	98629.966
e	0.2	52.0075	188804.87
e	0.5	51.7991	198432.87
e	0.8	0.134976	25000500
f	0.2	51.9639	187345.12
f	0.5	51.7923	189728.70
f	0.8	0.134976	25000500
g	0.2	51.9136	188723.45
g	0.5	51.9849	198332.46
g	0.8	0.157393	25005000
h	0.2	51.996	187188.33
h	0.5	51.8142	189567.04
h	0.8	51.9744	193418.46