Midterm
MKC523, Foundations of Solid Mechanics

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Question 1

1) The Levinson beam theory is based on the same displacement field as the Reddy-Bickford beam theory. As opposed to using the variationally-derived equations of equilibrium, Levinson (1981) used the thickness-integrated equations of elasticity which are exactly the same as those of the Timoshenko beam theory:

$$-\frac{dM_{xx}^L}{dx} + Q_x^L = 0, \quad -\frac{dQ_x^L}{dx} = q$$

The stress resultant-displacement relations for the Levinson beam theory are the same as those in Reddy-Bickford beam theory and they are

$$M_{xx}^L = \hat{D}_{xx}\frac{d\phi^L}{dx} - \alpha F_{xx}\frac{d^2 w_0^L}{dx^2}$$
$$Q_x^L = \hat{\mathrm{A}}_{xz}\left(\phi^L + \frac{dw_0^L}{dx}\right)$$

where the stiffnesses $\hat{D}_{xx}$, F_{xx} and $\hat{\mathrm{A}}_{xz}$ are defined as

$$\hat{D}_{xx} = D_{xx} - \alpha F_{xx}, \quad \hat{F}_{xx} = F_{xx} - \alpha H_{xx}$$
$$\hat{A}_{xz} = A_{xz} - \beta D_{xz}, \quad \hat{D}_{xz} = D_{xz} - \beta F_{xz}$$

$$(A_{xx}, D_{xx}, F_{xx}, H_{xx}) = \int_A (1, z^2, z^4, z^6) E_x \, dA$$
$$(A_{xz}, D_{xz}, F_{xz}) = \int_A (1, z^2, z^4) G_{xz} \, dA$$

Show that the relationships between the Levinson and Euler beam theories are as follows:

$$Q_x^L = Q_x^E + C_1$$
$$M_{xx}^L = M_{xx}^E + C_1 x + C_2$$
$$\phi^L = -\frac{dw_0^E}{dx} + \frac{\alpha F_{xx}}{D_{xx}\hat{\mathrm{A}}_{xz}}\left(Q_x^E + C_1\right) + \frac{1}{D_{xx}}\left(C_1\frac{x^2}{2} + C_2 x + C_3\right)$$
$$w_0^L = w_0^E + \frac{\hat{D}_{xx}}{D_{xx}\hat{\mathrm{A}}_{xz}}\left(M_{xx}^E + C_1 x\right) - \frac{1}{D_{xx}}\left(C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4\right)$$

$$(1)\quad M_{xx}^E = -D_{xx}\frac{d^2 w_0^E}{dx^2} \quad (1) \qquad M_{xx}^L = \hat{D}_{xx}\frac{d\phi^L}{dx} - \alpha F_{xx}\frac{d^2 w_0^L}{dx^2} \quad (3)$$

$$Q_x^E = \frac{dM_{xx}}{dx} = -\frac{d}{dx}\left(D_{xx}\frac{d^2 w_0^E}{dx^2}\right) \quad (2) \qquad Q_x^L = \hat{A}_{xz}\left(\phi^L + \frac{dw_0^L}{dx}\right) \quad (4)$$

Also, thickness integrated equations of elasticity are same in Euler and Levinson beam theories:

$$(5)\quad -\frac{dM_{xx}}{dx} + Q_x = 0 \quad , \quad -\frac{dQ_x}{dx} = q \quad (6)$$

Substituting (2) into (6) $\quad \frac{dQ_x^E}{dx} = \frac{d^2 M_{xx}^E}{dx^2} = -\frac{d^2}{dx^2}\left(D_{xx}\frac{d^2 w_0^E}{dx^2}\right)$

Also, by substituting (3) and (4) into (5) $\longrightarrow \frac{d\overbrace{M_{xx}}^{(3)}}{dx} = -\underbrace{Q_x}_{(4)}$

$$(7)\quad \hat{A}_{xz}\left(\phi^L + \frac{dw_0^L}{dx}\right) = \frac{d}{dx}\left(\hat{D}_{xx}\frac{d\phi^L}{dx} - \alpha F_{xx}\frac{d^2 w_0^L}{dx^2}\right) \quad \text{and} \quad \frac{d}{dx}\left(\hat{A}_{xz}\left(\phi^L + \frac{dw_0^L}{dx}\right)\right) = -q \quad (8)$$

differentiate

$$\frac{d^2}{dx^2}\underbrace{\left(\hat{D}_{xx}\frac{d\phi^L}{dx} - \alpha F_{xx}\frac{d^2 w_0^L}{dx^2}\right)}_{M_{xx}^L} = \frac{d}{dx}\underbrace{\left(\hat{A}_{xz} + \frac{dw_0^L}{dx}\right)}_{Q_x^L} \Rightarrow \frac{d^2 M_{xx}^L}{dx} = \frac{dQ_x^L}{dx} = -q \quad \text{and} \quad \frac{d^2 M_{xx}^E}{dx^2} = \frac{dQ_x}{dx} = -q$$

$$\Rightarrow Q_x^L = Q_x^E + C_1 , \quad (9)$$

$$M_{xx}^L = M_{xx}^E + C_1 x + C_2 \quad (10)$$

$$10 \rightarrow \hat{D}_{xx}\frac{d\phi^L}{dx} - \alpha F_{xx}\frac{d^2 w_0^L}{dx^2} = -D_{xx}\frac{d^2 w_0^E}{dx^2} + C_1 x + C_2 \qquad 9 \rightarrow \hat{A}_{xz}\left(\phi^L + \frac{dw_0^L}{dx}\right) = Q_x^E + C_1$$

integrate

$$\hat{D}_{xx}\frac{d\phi^L}{dx} = \alpha F_{xx}\frac{d^2 w_0^L}{dx^2} - D_{xx}\frac{d^2 w_0^E}{dx^2} + C_1 x + C_2 \qquad \frac{dw_0^L}{dx} = \frac{Q^E + C_1}{\hat{A}_{xz}} - \phi^L \quad (11)$$

$$\hat{D}_{xx}\phi^L = \alpha F_{xx}\frac{dw_0^L}{dx} - D_{xx}\frac{dw_0^E}{dx} + \frac{C_1 x^2}{2} + C_2 x + C_3 \qquad \hat{D}_{xx} = D_{xx} - \alpha F_{xx} \quad (12)$$

$$(11),(12) \rightarrow (D_{xx} - \cancel{\alpha F_{xx}})\phi^L = \frac{\alpha F_{xx}}{\hat{A}_{xz}}(Q^E + C_1) - \cancel{\alpha F_{xz}\phi^L} - D_{xx}\frac{dw_0^E}{dx} + \frac{C_1 x^2}{2} + C_2 + C_3$$

$$\phi^L = -\frac{dw_0^E}{dx} + \frac{\alpha F_{xz}}{D_{xx}\hat{A}_{xz}}(Q^E + C_1) + \frac{1}{D_{xx}}\left(\frac{C_1 x^2}{2} + C_2 x + C_3\right) \quad (13)$$

for w_0: $(11) \longrightarrow \phi^L = \frac{Q^E + C_1}{\hat{A}_{xz}} - \frac{dw_0^L}{dx}$ $\; : \; \frac{Q^E + C_1}{\hat{A}_{xz}} - \frac{dw_0^L}{dx} = -\frac{dw_0^E}{dx} + \frac{\alpha F_{xz}}{D_{xx}\hat{A}_{xz}}(Q^E + C_1) + \frac{1}{D_{xx}}\left(\frac{C_1 x^2}{2} + C_2 x + C_3\right)$

integrate

$$\frac{dw_0^L}{dx} = \frac{dw_0^E}{dx} + \frac{D_{xx}(Q^E + C_1)}{D_{xx}\hat{A}_{xz}} - \frac{\alpha F_{xz}}{D_{xx}\hat{A}_{xz}}(Q^E + C_1) - \frac{1}{D_{xx}}\left(\frac{C_1 x^2}{2} + C_2 x + C_3\right)$$

$$w_0^L = w_0^E + \frac{\hat{D}_{xx}}{D_{xx}\hat{A}_{xz}}(M_{xx}^E + C_1 x) - \frac{1}{D_{xx}}\left(\frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4\right)$$

2) Show that six compatibility equations given by Equations (1) may also be represented by the three independent fourth-order equations given by Equations (2).

$$\frac{\partial^2 \varepsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \varepsilon_{22}}{\partial x_1^2} = 2\frac{\partial^2 \varepsilon_{12}}{\partial x_1 \partial x_2} \quad (1)$$

$$\frac{\partial^2 \varepsilon_{22}}{\partial x_3^2} + \frac{\partial^2 \varepsilon_{33}}{\partial x_2^2} = 2\frac{\partial^2 \varepsilon_{23}}{\partial x_2 \partial x_3} \quad (2)$$

$$\frac{\partial^2 \varepsilon_{33}}{\partial x_1^2} + \frac{\partial^2 \varepsilon_{11}}{\partial x_3^2} = 2\frac{\partial^2 \varepsilon_{31}}{\partial x_3 \partial x_1} \quad (3)$$

$$\frac{\partial^2 \varepsilon_{11}}{\partial x_2 \partial x_3} = \frac{\partial}{\partial x_1}\left(-\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) \quad (4) \qquad \text{(Eqn.1)}$$

$$\frac{\partial^2 \varepsilon_{22}}{\partial x_3 \partial x_1} = \frac{\partial}{\partial x_2}\left(-\frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1}\right) \quad (5)$$

$$\frac{\partial^2 \varepsilon_{33}}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_3}\left(-\frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2}\right) \quad (6)$$

$$\frac{\partial^4 \varepsilon_{11}}{\partial x_2^2 \partial x_3^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(-\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) \quad (a)$$

$$\frac{\partial^4 \varepsilon_{22}}{\partial x_3^2 \partial x_1^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(-\frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1}\right) \quad (b) \qquad \text{(Eqn. 2)}$$

$$\frac{\partial^4 \varepsilon_{33}}{\partial x_1^2 \partial x_2^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(-\frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2}\right) \quad (c)$$

$\frac{\partial^2}{\partial x_3^2}$ for (1) $\rightarrow \frac{\partial^4 \varepsilon_{11}}{\partial x_2^2 \partial x_3^2} + \frac{\partial^4 \varepsilon_{22}}{\partial x_1^2 \partial x_3^2} = 2\frac{\partial^4 \varepsilon_{12}}{\partial x_1 \partial x_2 \partial x_3^2}$ (7)

$\frac{\partial^2}{\partial x_1^2}$ for (2) $\rightarrow \frac{\partial^4 \varepsilon_{22}}{\partial x_3^2 \partial x_1^2} + \frac{\partial^4 \varepsilon_{33}}{\partial x_2^2 \partial x_1^2} = 2\frac{\partial^4 \varepsilon_{23}}{\partial x_2 \partial x_3 \partial x_1^2}$ (8)

$\frac{\partial^2}{\partial x_2^2}$ for (3) $\rightarrow \frac{\partial^4 \varepsilon_{33}}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 \varepsilon_{11}}{\partial x_3^2 \partial x_2^2} = 2\frac{\partial^4 \varepsilon_{31}}{\partial x_3 \partial x_1 \partial x_2^2}$ (9)

(7) − (8) + (9) $\rightarrow \frac{\partial^4 \varepsilon_{11}}{\partial x_2^2 \partial x_3^2} + \cancel{\frac{\partial^4 \varepsilon_{22}}{\partial x_1^2 \partial x_3^2}} - \cancel{\frac{\partial^4 \varepsilon_{22}}{\partial x_3^2 \partial x_1^2}} - \cancel{\frac{\partial^4 \varepsilon_{33}}{\partial x_2^2 \partial x_1^2}} + \cancel{\frac{\partial^4 \varepsilon_{33}}{\partial x_1^2 \partial x_2^2}} + \frac{\partial^4 \varepsilon_{11}}{\partial x_3^2 \partial x_2^2} = \frac{2\partial^4 \varepsilon_{12}}{\partial x_1 \partial x_2 \partial x_3^2} - \frac{2\partial^4 \varepsilon_{23}}{\partial x_2 \partial x_3 \partial x_1^2} - \frac{2\partial^4 \varepsilon_{31}}{\partial x_3 \partial x_1 \partial x_2^2}$

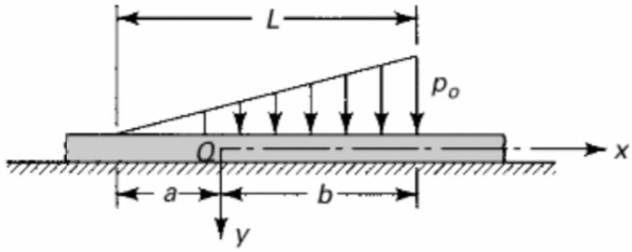
$\rightarrow \frac{\partial^4 \varepsilon_{11}}{\partial x_2^2 \partial x_3^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(\frac{-\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) = (a)$; simplification $\frac{\partial \varepsilon_{11}}{\partial x_2 \partial x_3} = \frac{\partial}{\partial x_1}\left(-\frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3}\right) \rightarrow (4)$

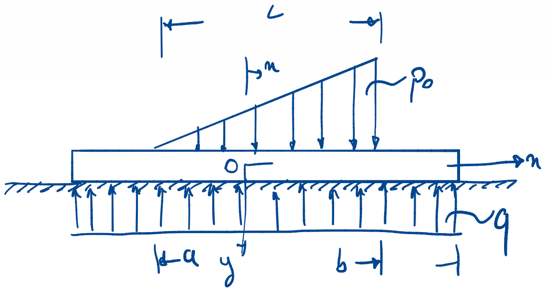
(7) + (8) − (9) $\rightarrow \frac{\partial^4 \varepsilon_{22}}{\partial x_3^2 \partial x_1^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(\frac{-\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1}\right) = (b) \rightarrow \frac{\partial \varepsilon_{22}}{\partial x_3 \partial x_1} = \frac{\partial^2}{\partial x_2}\left(\frac{-\partial \varepsilon_{31}}{\partial x_2} + \frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1}\right) \rightarrow (5)$

−(7) + (8) + (9) $\rightarrow \frac{\partial^4 \varepsilon_{33}}{\partial x_1^2 \partial x_2^2} = \frac{\partial^3}{\partial x_1 \partial x_2 \partial x_3}\left(\frac{-\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2}\right) = (c) \rightarrow \frac{\partial \varepsilon_{33}}{\partial x_1 \partial x_2} = \frac{\partial^3}{\partial x_3}\left(-\frac{\partial \varepsilon_{12}}{\partial x_3} + \frac{\partial \varepsilon_{23}}{\partial x_1} + \frac{\partial \varepsilon_{31}}{\partial x_2}\right) \rightarrow (6)$

Question 3

3) Determine the deflection at any point O under the triangular loading acting on an infinite beam on an elastic foundation.

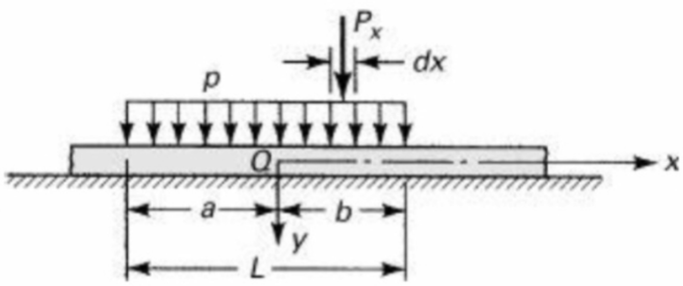




Load in terms of x ; $p = \frac{p_o}{L}(a-x)$ for $x<0$

$p = \frac{p_o}{L}(a+x)$ for $x>0$

General solution : $v = e^{-\beta x}(C\cos\beta x + D\sin\beta x)$ (eqn. 9.5 in lecture notes)



(from lecture notes)

for infinite beam on an elastic foundation and loaded at the origin

$$v = \frac{p\beta}{2k} e^{-\beta x}(\cos\beta x + \sin\beta x) \quad (9.6a)$$

$$v = \frac{p\beta}{2k} f_1$$

for $P_x = \frac{p_o}{L} dx$

$$\Delta v = \frac{p_o dx}{2k}\beta e^{-\beta x}(\cos\beta x + \sin\beta x)$$

$$\Rightarrow v_\theta = \int_0^a \frac{p_o(a-1)dx}{2kL}\beta e^{-\beta x}(\cos\beta x + \sin\beta x) + \int_0^b \frac{p_o(a+1)dx}{2kL}\beta e^{-\beta x}(\cos\beta x + \sin\beta x)$$

$$v_\theta = \frac{p_o}{2kL}\left(\int_0^a (a-1)\beta e^{-\beta x}(\cos\beta x + \sin\beta x)dx + \int_0^b (a+1)\beta e^{-\beta x}(\cos\beta x + \sin\beta x)dx\right)$$

$$v_\theta = \frac{p_o}{2kL}\left(\int_0^a (a\beta e^{-\beta x}\cos\beta x + a\beta e^{-\beta x}\sin\beta x - \beta e^{-\beta x}\cos\beta x - \beta e^{-\beta x}\sin\beta x)dx + \int_0^b a\beta e^{-\beta x}\cos\beta x + a\beta e^{-\beta x}\sin\beta x + \beta e^{-\beta x}\cos\beta x + \beta e^{-\beta x}\sin\beta x dx\right.$$

$$v_\theta = \frac{p_o\beta}{2kL}\left(\int_0^a af_4 + af_2 - f_4 - f_2\,dx + \int_0^b af_4 + af_2 + f_4 + f_2\,dx\right.$$

$$v_\theta = \frac{p_o\beta}{2kL}\left[\int_0^a (a\cdot f_1 - f_4 - f_2)dx + \int_0^b (a\cdot f_1 + f_1)dx = \frac{p_o\beta}{2kL}\left[\left(-\frac{a}{\beta}f_4 + \frac{1}{2\beta}f_3 - \frac{1}{2\beta}f_1\right)\Big|_0^a + \left(-\frac{a}{\beta}f_4 + \frac{f_4}{\beta}\right)\Big|_0^b\right]\right.$$

$$v_\theta = \frac{p_o\beta}{2kL}\left[-\frac{a}{\beta}f_4(\beta a) - \frac{a}{\beta} + \frac{1}{2\beta}f_3(\beta a) + \cancel{\frac{1}{2\beta}} - \frac{1}{2\beta}f_1(\beta a) + \cancel{\frac{1}{2\beta}} - \frac{a}{\beta}f_4(\beta b) - \frac{a}{\beta} + \frac{f_4(\beta b)}{\beta} + \cancel{\frac{1}{\beta}}\right]$$

$$v_\theta = \frac{p_0}{2kL}\left[-a f_4(\beta a) - a + \frac{1}{2} f_3(\beta a) - \frac{1}{2} f_1(\beta a) - a f_4(\beta b) - a + f_4(\beta b)\right] \quad , \quad f_3 - f_1 = 2f_2$$

$$v_\theta = \frac{p_0}{2kL}\left[-a f_4(\beta a) + (1-a)\cdot f_4(\beta b) + f_2(\beta a) - 2a\right]$$

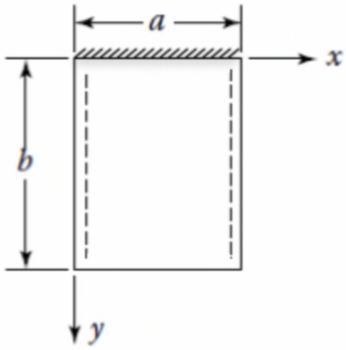
4) The total potential energy functional for the bending of circular plates on linear elastic foundation is given by

$$\Pi(w) = \frac{D}{2}\int_{\Omega}\left[\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)^2 - 2(1-\nu)\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + 2(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta}\right)^2\right] r\,dr\,d\theta$$

$$+\frac{1}{2}\int_{\Omega} k w^2 r\,dr\,d\theta - \int_{\Omega} q w r\,dr\,d\theta$$

where D and ν are elastic constants, and w is the deflection (in the z direction) of the center-plane of the plate, q is the distributed loading function and k is the modulus of the foundation. <u>Derive the Euler equation and natural boundary conditions of the functional given for this problem.</u>

5) Consider a rectangular plate with two opposite sides (x = 0 and x = a) simply supported; the third edge (y = 0) is built-in, and the fourth edge (y = b) is free. The plate is subjected to a uniform pressure of intensity p_0.



a) Retaining only the first three terms of the **Levy's solution***, determine the deflection at the midpoint of the free edges and the bending moments at the midpoint of the clamped edge.

b) Find the maximum values of stresses σ_x and σ_y at the center of the plate?

(Take a = 2m; b =3:5m, h = 0:15m; E = 210 GPa; ν = 0:3 for numerical results.)

*While Navier's solution is double series solution which may not be preferred due to slow convergence of the series, Levy's solution is single Fourier series solution to rectangular plates with easier numerical calculations. Levy suggested that the solution of the non-homogenous biharmonic governing plate equation can be expressed in terms of complementary w_h and particular w_p, each of which consists of a single Fourier series where the unknown functions are determined from the prescribed boundary conditions.

$$w = w_h + w_p$$

$$w_h = \sum_{m=1}^{\infty} f_m(y)\sin\frac{m\pi x}{a}$$

$$w_p = \sum_{m=1}^{\infty} g_m(y)\sin\frac{m\pi x}{a}$$

Given: $M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$, $V_y = Q_y + \frac{\partial M_{xy}}{\partial x} = -D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right]$

$\sigma_x = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$, $\sigma_y = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$

from Ventsel, Thin Plates and Shells: Theory: Analysis, and applications

homogeneous solution

$w = w_h + w_p$, $\left.\frac{\partial^2 w}{\partial x^2} = 0\right|_{x=0,\, x=a}$, secondary boundary condition.

particular solution

$w_h = \sum_{m=1}^{\infty} f_m(y)\sin\frac{m\pi x}{a}$, $\nabla^4 w = 0$ $\Rightarrow \underbrace{\left[\left(\frac{m\pi}{a}\right)^4 f_m(y) - 2\left(\frac{m\pi}{a}\right)^2\frac{d^2 f_m(y)}{dy^2} + \frac{d^4 f_m(y)}{dy^4}\right]}_{=0}\sin\frac{m\pi x}{a} = 0$, $f_m(y) = e^{\lambda y}$

$\Rightarrow$ Characteristic equation: $\lambda^4 - \frac{2m^2\pi^2}{a^2}\lambda^2 + \frac{m^4\pi^4}{a^4} = 0$; $\lambda_{1,2} = \frac{m\pi}{a}$, $\lambda_{3,4} = -\frac{m\pi}{a}$

$\Rightarrow f_m(y) = A'_m e^{\frac{m\pi y}{a}} + B'_m e^{-\frac{m\pi y}{a}} + \frac{m\pi y}{a}\left(C'_m e^{\frac{m\pi y}{a}} + D'_m e^{-\frac{m\pi y}{a}}\right)$

Using hyperbolic functions: $f_m(y) = A_m \sinh\frac{m\pi y}{a} + B_m\cosh\frac{m\pi y}{a} + \frac{m\pi y}{a}\left(C_m\sinh\frac{m\pi y}{a} + D_m\cosh\frac{m\pi y}{a}\right)$

$$w_h = \sum_{m=1}^{\infty} \left[A_m \sinh\frac{m\pi y}{a} + B_m \cosh\frac{m\pi y}{a} + \frac{m\pi y}{a}\left(C_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a} \right) \right] \sin\frac{m\pi x}{a}$$

$$w_p(x,y) = \sum_{m=1}^{\infty} g_m(y) \sin\frac{m\pi x}{a} \quad ; \quad p_m(y) = \frac{2}{a}\int_0^a p(x,y) \sin\frac{m\pi x}{a}\,dx$$ → for uniform pressure with intensity p_0 : $p_m(y) = \frac{2p_0}{m\pi}(1-\cos m\pi)$

which satisfies diferantial equation : $\frac{d^4 g_m(y)}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 g_m(y)}{dy^2} + \left(\frac{m\pi}{a}\right)^4 g_m(y) = \frac{p_m(y)}{D}$

$\Rightarrow p_1(y) = \frac{4p_0}{\pi}$, $p_2(y) = 0$, $p_3(y) = \frac{4p_0}{3\pi}$ ... ; for $m = 1, 2, 3 \ldots$

This also means that the particular solution $g_m(y) = $ constant $\Rightarrow \left(\frac{m\pi}{a}\right)^4 g_m(y) = \frac{p_m(y)}{D}$; $g_m(y) = \left(\frac{a}{m\pi}\right)^4 \cdot \frac{2p_0}{m\pi}(1-\cos m\pi)$

$g_m(y) = \left(\frac{a}{m\pi}\right)^4 \frac{p_m(y)}{D} \Rightarrow g_1(y) = \frac{4a^4 p_0}{\pi^5 D}$, $g_2(y) = 0$, $g_3(y) = \frac{4a^4 p_0}{243\pi^5 D}$

$$w = \sum_{m=1}^{\infty} \left(\frac{2 \cdot a^4 \cdot p_0 \cdot (1-\cos m\pi)}{(m\pi)^5} + A_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a} + \frac{m\pi y}{a} \cdot \left(C_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a} \right) \right) \cdot \sin\frac{m\pi x}{a}$$

① $w\big|_{y=0} = 0$, ② $\frac{dw}{dy}\big|_{y=0} = 0$, ③ $M_y\big|_{y=0} = 0$, ④ $V_y\big|_{y=0} = 0$

① : $B_m = -\frac{2 \cdot a^4 p_0 \cdot (1-\cos m\pi)}{(m\pi^5)}$

② ; $\frac{\partial w}{\partial y} = \sum_{m=1}^{\infty} \frac{m \cdot \pi}{a} \cdot \left(A_m \cosh\frac{m\pi y}{a} + B_m \sinh\frac{m\pi y}{a} + \left(C_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a} \right) + \frac{m\pi y}{a} \cdot \left(C_1 \cosh\frac{m\pi y}{a} + D_m \sinh\frac{m\pi y}{a} \right) \right) \sin\frac{m\pi x}{a}$

$A_m = D_m$

③ : $M_y = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + v \frac{\partial^2 w}{\partial x^2} \right) = D \cdot \sum_{m=1}^{\infty} \left(\frac{m\pi}{a} \right)^2 \cdot \left((v-1)\left(A_m \sinh\frac{m\pi y}{a} + B_m \cosh\frac{m\pi y}{a} + \frac{m\pi y}{a} \cdot \left(C_1 \sinh\frac{m\pi y}{a} + D_1 \cosh\frac{m\pi y}{a} \right) - 2\left(C_1 \cosh\frac{m\pi y}{a} \ldots \right.\right.\right.$

$+ D_1 \sinh\frac{m\pi y}{a} + \frac{2 \cdot a^4 \cdot p_0 \cdot (1-\cos m\pi)}{(m\pi)^5} \Big) \sin\frac{m\pi x}{a}$